Exercise 5A

1 a $(x_1, y_1) = (4, 2), (x_2, y_2) = (6, 3)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3-2}{6-4}$$

$$= \frac{1}{2}$$

b $(x_1, y_1) = (-1, 3), (x_2, y_2) = (5, 4)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4-3}{5-(-1)}$$

$$= \frac{1}{6}$$

c $(x_1, y_1) = (-4, 5), (x_2, y_2) = (1, 2)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2-5}{1-(-4)}$$

$$= -\frac{3}{5}$$

d $(x_1, y_1) = (2, -3), (x_2, y_2) = (6, 5)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-3)}{6-2}$$

$$= \frac{8}{4}$$

$$= 2$$

e $(x_1, y_1) = (-3, 4), (x_2, y_2) = (7, -6)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-4}{7-(-3)}$$

$$= \frac{-10}{10}$$

$$= -1$$

f $(x_1, y_1) = (-12, 3), (x_2, y_2) = (-2, 8)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{8-3}{-2-(-12)}$$

$$= \frac{1}{2}$$

g $(x_1, y_1) = (-2, -4), (x_2, y_2) = (10, 2)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-4)}{10-(-2)}$$

$$= \frac{6}{12}$$

$$= \frac{1}{2}$$

h $(x_1, y_1) = \left(\frac{1}{2}, 2\right), (x_2, y_2) = \left(\frac{3}{4}, 4\right)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{\frac{3}{4}-\frac{1}{2}}$$

$$= \frac{2}{\frac{1}{4}}$$

$$= 8$$

i $(x_1, y_1) = \left(\frac{1}{4}, \frac{1}{2}\right), (x_2, y_2) = \left(\frac{1}{2}, \frac{2}{3}\right)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{2}{3}-\frac{1}{2}}{\frac{1}{2}-\frac{1}{4}}$$

$$= \frac{\frac{1}{6}}{\frac{1}{4}}$$

$$= \frac{2}{3}$$

j $(x_1, y_1) = (-2.4, 9.6), (x_2, y_2) = (0, 0)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0-9.6}{0-(-2.4)}$$

$$= \frac{-9.6}{2.4}$$

$$= -4$$

k $(x_1, y_1) = (1.3, -2.2), (x_2, y_2) = (8.8, -4.7)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-4.7-(-2.2)}{8.8-1.3}$$

1 k $\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{-2.5}{7.5}$

$=-\dfrac{1}{3}$

l $(x_1, y_1)=(0, 5a), (x_2, y_2)=(10a, 0)$

$\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{0-5a}{10a-0}$

$=\dfrac{-5a}{10a}$

$=\dfrac{-5}{10}$

$=-\dfrac{1}{2}$

m $(x_1, y_1)=(3b, -2b), (x_2, y_2)=(7b, 2b)$

$\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{2b-(-2b)}{7b-3b}$

$=\dfrac{4b}{4b}$

$=1$

n $(x_1, y_1)=(p, p^2), (x_2, y_2)=(q, q^2)$

$\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{q^2-p^2}{q-p}$

$=\dfrac{(q-p)(q+p)}{q-p}$

$=q+p$

2 $(x_1, y_1)=(3, -5), (x_2, y_2)=(6, a)$

$\dfrac{y_2-y_1}{x_2-x_1}=4$

So $\dfrac{a-(-5)}{6-3}=4$

$\Rightarrow \dfrac{a+5}{3}=4$

$\Rightarrow a+5=12$

$\Rightarrow a=7$

3 $(x_1, y_1)=(5, b), (x_2, y_2)=(8, 3)$

$\dfrac{3-b}{8-5}=-3$

$\dfrac{3-b}{3}=-3$

$3-b=-9$

$b=12$

4 $(x_1, y_1)=(c, 4), (x_2, y_2)=(7, 6)$

$\dfrac{6-4}{7-c}=\dfrac{3}{4}$

$\dfrac{2}{7-c}=\dfrac{3}{4}$

$2=\dfrac{3}{4}(7-c)$

$8=3(7-c)$

$8=21-3c$

$-13=-3c$

$c=\dfrac{-13}{-3}$

$=\dfrac{13}{3}$

$=4\dfrac{1}{3}$

5 $(x_1, y_1)=(-1, 2d), (x_2, y_2)=(1, 4)$

$\dfrac{4-2d}{1-(-1)}=-\dfrac{1}{4}$

$\dfrac{4-2d}{2}=-\dfrac{1}{4}$

$2-d=-\dfrac{1}{4}$

$2\dfrac{1}{4}-d=0$

$d=2\dfrac{1}{4}$

6 $(x_1, y_1) = (-3, -2), (x_2, y_2) = (2e, 5)$

$$\frac{5-(-2)}{2e-(-3)} = 2$$

$$\frac{7}{2e+3} = 2$$

$$7 = 2(2e+3)$$

$$7 = 4e+6$$

$$4e = 1$$

$$e = \frac{1}{4}$$

7 $(x_1, y_1) = (7, 2), (x_2, y_2) = (f, 3f)$

$$\frac{3f-2}{f-7} = 4$$

$$3f-2 = 4(f-7)$$

$$3f-2 = 4f-28$$

$$-2 = f-28$$

$$28-2 = f$$

$$f = 26$$

8 $(x_1, y_1) = (3, -4), (x_2, y_2) = (-g, 2g)$

$$\frac{2g-(-4)}{-g-3} = -3$$

$$\frac{2g+4}{-g-3} = -3$$

$$2g+4 = -3(-g-3)$$

$$2g+4 = 3g+9$$

$$4 = g+9$$

$$g = -5$$

9 The gradient of AB is:

$$\frac{4-3}{4-2} = \frac{1}{2}$$

The gradient of AC is:

$$\frac{7-3}{10-2} = \frac{4}{8}$$

$$= \frac{1}{2}$$

The gradients are equal and there is a point in common between the two line segments so the points can be joined by a straight line.

10 If the points $A(-2a, 5a)$, $B(0, 4a)$ and $C(6a, a)$ are collinear, then they all lie on the same straight line.

The gradient of AB is:

$$\frac{4a-5a}{0-(-2a)} = \frac{-a}{2a}$$

$$= -\frac{1}{2}$$

The gradient of AC is:

$$\frac{a-5a}{6a-(-2a)} = \frac{-4a}{8a}$$

$$= -\frac{1}{2}$$

The gradients are both $-\frac{1}{2}$ and there is a point in common between the two line segments so the points are collinear.